

1 Ejercicios de límites Ficha I

Calcula los siguientes límites

1. $\lim_{x \rightarrow \frac{1}{2}} \frac{x^2 - 5x + 2}{x^2 + 3x - 3}$ $\frac{1}{5}$
2. $\lim_{x \rightarrow 4} \frac{2x^2 - 7x - 4}{x^3 + x^2 - 10}$ 0
3. $\lim_{x \rightarrow 3^+} \frac{x + 2}{x^2 - 4x + 3}$ $+\infty$
4. $\lim_{x \rightarrow 3^-} \frac{x + 2}{x^2 - 4x + 3}$ $-\infty$
5. $\lim_{x \rightarrow 3} \frac{x + 2}{x^2 - 4x + 3}$ No existe ⁽¹⁾
6. $\lim_{x \rightarrow 3} \frac{x^2 + 5}{x^2 - 6x + 9}$ $+\infty$.⁽²⁾
7. $\lim_{x \rightarrow -1} \frac{x + 1}{x^3 + 3x^2 + 3x + 1}$ $+\infty$
8. $\lim_{x \rightarrow -2} \frac{x^2 + 4x + 4}{x^3 + 6x^2 + 12x + 8}$. No existe
 Ayuda: $\begin{cases} x^2 + 4x + 4 = (x + 2)^2 \\ x^3 + 6x^2 + 12x + 8 = (x + 2)^3 \end{cases}$
9. $\lim_{x \rightarrow 3} \sqrt{\frac{x^3 - 2x + 5}{4x^2 - x + 2}}$ $\sqrt{\frac{26}{35}}$
10. $\lim_{x \rightarrow 5} \frac{x^2 - 25}{10 - 2x}$ -5
11. $\lim_{x \rightarrow 3} \frac{(x - 1)(x - 3)^3}{(2x + 1)(2x - 1)(x - 3)^2}$ 0
12. $\lim_{x \rightarrow 4} \frac{x^2 - 16}{\sqrt{2x + 1} - 3}$ 24
13. $\lim_{x \rightarrow 1} \frac{\sqrt{x + 1} - 2}{\sqrt{2x + 10} - \sqrt{5x + 1}}$ $-\frac{1}{3}\sqrt{3}$
14. $\lim_{x \rightarrow 3} \frac{\sqrt{x + 1} - 2}{\sqrt{2x + 10} - \sqrt{5x + 1}}$ $-\frac{2}{3}$

¹ $\lim_{x \rightarrow 3^+} \frac{x + 2}{x^2 - 4x + 3} = +\infty$ y $\lim_{x \rightarrow 3^-} \frac{x^2 + 5}{x^2 - 6x + 9} = -\infty$
² $x^2 - 6x + 9 = (x - 3)^2$

15. $\lim_{x \rightarrow -a} \frac{x^5 + a^5}{x^4 - a^4} \qquad -\frac{5}{4}a$
16. $\lim_{h \rightarrow 0} \frac{(x+h)^4 - x^4}{h} \qquad 4x^3$
17. $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x^3 - 2x^2 + 4x - 8} \qquad \frac{1}{2}$
18. $\lim_{x \rightarrow -2} \frac{2x^3 + 9x^2 + 12x + 4}{3x^3 + 11x^2 + 8x - 4} \qquad \frac{3}{7}$
19. $\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x} \qquad \frac{1}{4}$
20. $\lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \qquad \frac{1}{2\sqrt{x}}$
21. $\lim_{x \rightarrow 1} \frac{\sqrt[3]{x^2 - 1}}{\sqrt[6]{x^3 - 3x + 2}} \qquad \sqrt[6]{\frac{4}{3}}$
22. $\lim_{x \rightarrow 5} \frac{2x - 10}{\sqrt{x+4} - 3} \qquad 12$
23. $\lim_{x \rightarrow 2} \frac{x^2 + 2x - 8}{x^3 - 6x^2 + 11x - 6} \qquad -6$
24. $\lim_{x \rightarrow 0} \frac{x^2 - x}{5x^2 + 3x} \qquad -\frac{1}{3}$
25. $\lim_{x \rightarrow 2} \frac{x^3 - 4x^2 + 4x}{x^3 - 3x^2 + 4} \qquad \frac{2}{3}$
26. $\lim_{x \rightarrow 2} \left[\frac{x^2 - 13x + 22}{x^2 - 8x + 12} \right]^{\frac{x-1}{2}} \qquad \frac{3}{2}$
27. $\lim_{x \rightarrow 3} \frac{\sqrt{\frac{x}{3}} - 1}{x - 3} \qquad \frac{1}{6}$
28. $\lim_{x \rightarrow 3} \frac{x - \sqrt{6+x}}{x - 3} \qquad \frac{5}{6}$
29. $\lim_{x \rightarrow -1} \frac{x^4 - 1}{x^3 + 1} \qquad -\frac{4}{3}$
30. $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{\sqrt{x+3} - 2} \qquad 12$
31. $\lim_{x \rightarrow 3} e^{\frac{2x-4}{x^2-4}} \qquad \sqrt{e}$

$$32. \lim_{x \rightarrow 2} \frac{\sqrt[3]{x+6} - 2}{-x^2 + 3x - 2} \quad -\frac{1}{12}$$

Este límite presenta la indeterminación $\frac{0}{0}$

$$\text{Ayuda: Como } A-B = \frac{A^3 - B^3}{A^2 + AB + B^2} \text{ entonces: } \sqrt[3]{x+6} - 2 = \frac{(\sqrt[3]{x+6})^3 - 2^3}{(\sqrt[3]{x+6})^2 + 2\sqrt[3]{x+6} + 4}$$

$$\lim_{x \rightarrow 2} \frac{\sqrt[3]{x+6} - 2}{-x^2 + 3x - 2} = \lim_{x \rightarrow 2} \frac{(\sqrt[3]{x+6})^3 - 2^3}{\left[(\sqrt[3]{x+6})^2 + 2\sqrt[3]{x+6} + 4 \right] (-x^2 + 3x - 2)} =$$

$$\lim_{x \rightarrow 2} \frac{x-2}{\left[(\sqrt[3]{x+6})^2 + 2\sqrt[3]{x+6} + 4 \right] (-x+1)(x-2)} =$$

$$\lim_{x \rightarrow 2} \frac{1}{\left[(\sqrt[3]{x+6})^2 + 2\sqrt[3]{x+6} + 4 \right] (-x+1)} = \frac{1}{(4+4+4)(-1)} = -\frac{1}{12}$$

$$33. \lim_{x \rightarrow -2} \frac{\sqrt[3]{3x-2} + 2}{-x^2 + 3x + 10}$$

Este límite presenta la indeterminación $\frac{0}{0}$

$$\text{Al ser } A+B = \frac{A^3 + B^3}{A^2 - AB + B^2} \text{ entonces: } \sqrt[3]{3x-2} + 2 = \frac{(\sqrt[3]{3x-2})^3 + 2^3}{(\sqrt[3]{3x-2})^2 - 2(\sqrt[3]{3x-2}) + 4}$$

$$\lim_{x \rightarrow -2} \frac{\sqrt[3]{3x-2} + 2}{-x^2 + 3x + 10} = \lim_{x \rightarrow -2} \frac{(\sqrt[3]{3x-2})^3 + 2^3}{\left((\sqrt[3]{3x-2})^2 - 2\sqrt[3]{3x-2} + 4 \right) (-x^2 + 3x + 10)} =$$

$$\lim_{x \rightarrow -2} \frac{3(x+2)}{\left[(\sqrt[3]{3x-2})^2 - 2\sqrt[3]{3x-2} + 4 \right] (x+2)(-x+5)} =$$

$$\lim_{x \rightarrow -2} \frac{3}{\left[(\sqrt[3]{3x-2})^2 - 2\sqrt[3]{3x-2} + 4 \right] (-x+5)} = \frac{3}{(4+4+4)7} = \frac{1}{28}$$

$$34. \lim_{x \rightarrow 1} \frac{\sqrt[3]{x} - 1}{-x^2 + 1} \quad -\frac{1}{6}$$

$$35. \lim_{x \rightarrow -1} \frac{\sqrt[3]{x} + 1}{-x^2 + 1} \quad \frac{1}{6}$$

$$36. \lim_{x \rightarrow 5} \frac{\sqrt[5]{7x-3} - 2}{-x^2 + 25} \quad -\frac{7}{800}$$

$$\text{Ayuda: } A - B = \frac{A^5 - B^5}{A^4 + A^3B + A^2B^2 + AB^3 + B^4}$$

$$37. \lim_{x \rightarrow -5} \frac{\sqrt[5]{7x+3} + 2}{-x^2 + 25}$$

$${}^3A + B = \frac{A^5 + B^5}{A^4 - A^3B + A^2B^2 - AB^3 + B^4}$$